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Examples

1 Problems

1. Let $X \sim$ Bernoulli(p) and ϕ be the (signed) measure corresponding to the distribution function P_X.

   (a) Find the Lebesgue decomposition of ϕ with respect to the counting measure.

   (b) Find the Lebesgue decomposition of ϕ with respect to the Lebesgue measure.

   (c) Find the Lebesgue decomposition of ϕ with respect to the Dirac measure δ_1.

2. Let $X \sim$ Poisson(λ) and $Y_1, Y_2, \cdots \sim$ Gamma(α, β) random variables. Consider the random variable $Z = \sum_{i=0}^{X} Y_i$ and let ϕ be the (signed) measure corresponding to the distribution function F_Z.

   (a) Find the Lebesgue decomposition of ϕ with respect to the Lebesgue measure.

   (b) Find the Lebesgue decomposition of ϕ with respect to the Dirac measure δ_0.

3. Let $X \sim$ Exponential(θ) and let $Y \sim$ Bernoulli(p) random variables. Consider the random variable $Z = (-1)^Y \cdot X$ and let ϕ be the (signed) measure corresponding to the distribution function F_Z.

   (a) Find the Lebesgue decomposition of ϕ with respect to measure μ that satisfies $\mu((-\infty, 0)) = 0$ and $\mu([0, \infty)) = 1$.

   (b) Find the Lebesgue decomposition of ϕ with respect to measure μ that satisfies $\mu((-\infty, 0]) = 1$ and $\mu((0, \infty)) = 0$.

4. Let $\Omega = \{-1, 1\}$ and consider the measurable space $(\Omega, 2^\Omega)$. As well, let ϕ be the signed measure that satisfies $\phi(\{-1\}) = -1$ and $\phi(\{1\}) = 1$.

   (a) Find the Lebesgue decomposition of ϕ with respect to the Dirac measure δ_1.

   (b) Find the Lebesgue decomposition of ϕ with respect to the Dirac measure δ_{-1}.

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2 Solutions

1. Note that $\Omega = \{0, 1\}$ and the σ-field in question is 2^{Ω}. Moreover, ϕ and the measures in (a-c) are defined on the measurable space $(\Omega, 2^{\Omega})$.
   (a) Verify that $\phi_{ac} = \phi$ and ϕ_s is the zero measure.
   (b) Verify that $\phi_s = \phi$ and ϕ_{ac} is the zero measure.
   (c) Verify that $\phi_{ac} = \delta_1$ and $\phi_s = \delta_0$.
2. See "Tweedie Index Parameter".
   (a) The random variable Z has a point mass at zero and has continuous density on the positive reals. Verify that $\phi_s = \delta_0 \cdot \exp\{-\lambda\}$ and $\phi_{ac} = \phi - \phi_s$.
   (b) Verify that $\phi_{ac} = \delta_0 \cdot \exp\{-\lambda\}$ and $\phi_s = \phi - \phi_{ac}$.
3. See double exponential random variable article.
   (a) With respect to this measure, we bifurcate the double exponential random variable at 0. Be cautious in handling the boundary.
   (b) Reverse of (a).
4. (a) Verify that $\phi_{ac} = \delta_1$ and $\phi_s = -\delta_{-1}$.
   (b) Verify that $\phi_{ac} = -\delta_{-1}$ and $\phi_s = \delta_1$.