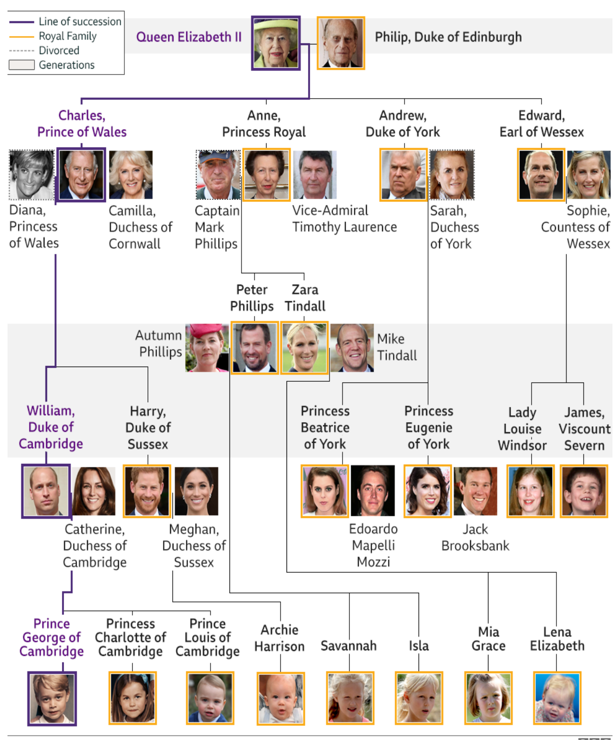
Pedigree Practice

BIOST 550
April 22, 2022

Let M_A be the mother of A, F_A be the father of A, $n(\cdot)$ be the number of meioses in a path, $\psi(\cdot,\cdot)$ be the kinship coefficient, and $f_A = \psi(M_A, F_A)$ be the inbreeding coefficient for A. Throughout the following exercises, we use the path-counting formula.

$$\sum_{A \in \mathcal{A}} \sum_{\mathcal{P}(A)} (1 + f_A)(1/2)^{n(\mathcal{P}(A))+1}$$

1 British Royal Family



1. Compute the kinship between Lady Louise Windsor and Prince George of Cambridge.

   (a)

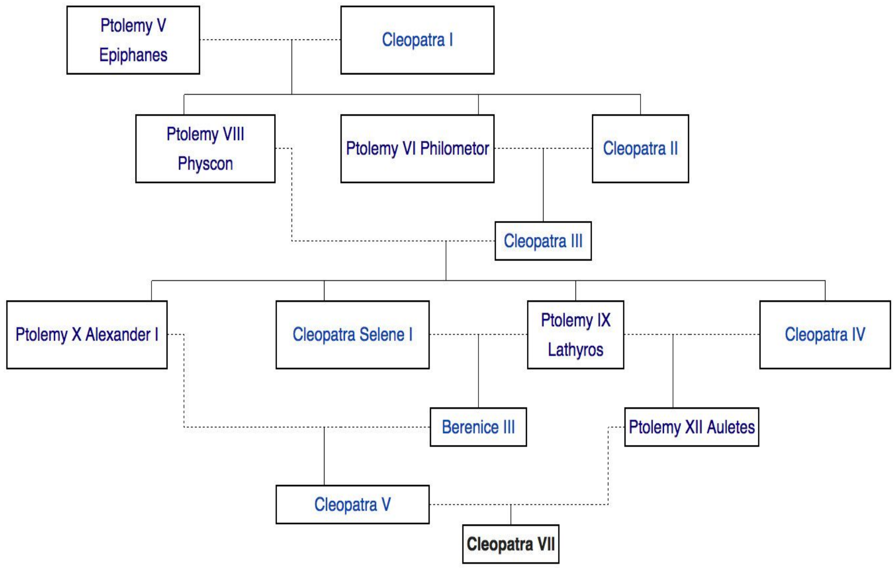
2. Compute the kinship of Archie Harrison and Edward, Earl of Wessex.

   (a)

3. Compute the kinship of Lena Elizabeth and Charles, Prince of Wales.

   (a)

Pedigree Practice



2 Cleopatra of Egypt

1. Compute the inbreeding coefficient of Cleopatra III.

   (a)

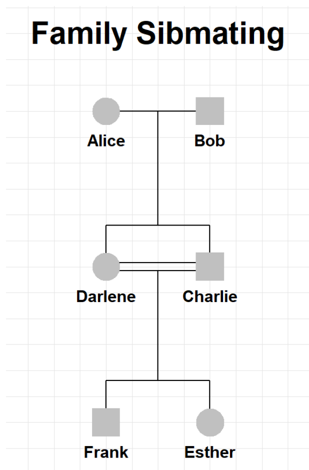
2. Compute the inbreeding coefficient of Cleopatra IV.

   (a)

3. Compute the inbreeding coefficient of Berenice III.

   (a)

3 Jacquard's Δ values



Consider the figure above. Compute Δ_i values for Charlie and Frank.

1. Δ_1

2. Δ_2

3. Δ_3

4. Δ_9

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Royal Pedigrees

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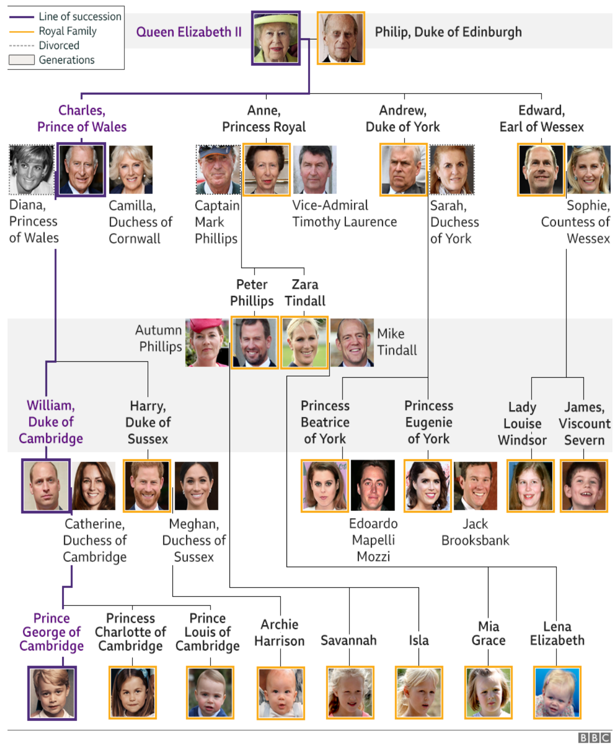
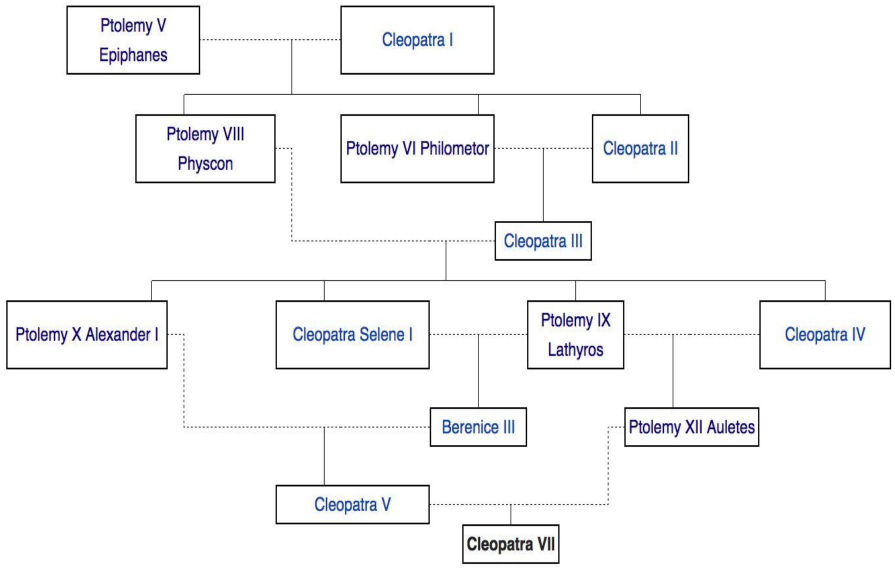
1 British Royal Family



1. Compute the kinship between Lady Louise Windsor and Prince George of Cambridge.
    (a) $2(1 + 0)(1/2)^{5+1} = 0.03125$. This is consistent with first cousins once removed.
2. Compute the kinship of Archie Harrison and Edward, Earl of Wessex.
    (a) $2(1 + 0)(1/2)^{4+1} = .0625$. This is consistent with a granduncle-grandnephew.
3. Compute the kinship of Lena Elizabeth and Charles, Prince of Wales.
    (a) $2(1 + 0)(1/2)^{4+1} = .0625$. This is consistent with a granduncle-grandniece.



2 Cleopatra of Egypt

1. Compute the inbreeding coefficient of Cleopatra III.

   (a) The kinship between Ptolemy VI Philometer and Cleopatra II is $2(1+0)(1/2)^{2+1} = 0.25$. This is consistent with siblings. Therefore, the inbreeding coefficient for Cleopatra III is 0.25.

2. Compute the inbreeding coefficient of Cleopatra IV.

   (a) The kinship between Ptolemy VIII Physcon and Cleopatra III is

   $$2(2)(1+0)(1/2)^{3+1} = 0.25.$$

   This is an uncle-niece pair twice over. Therefore, the inbreeding coefficient for Cleopatra IV is 0.25.

3. Compute the inbreeding coefficient of Berenice III.

   (a) The kinship between Cleopatra Selene I and Ptolemy IX Lathyros is

   $$1(1)(1+0)(1/2)^{2+1} + 1(1)(1+0.25)(1/2)^{2+1} + 2(4)(1+0)(1/2)^{5+1} = 0.40625.$$

   The first term is for the path through the non-inbred Ptolemy VIII Physcon, the second term is for the path through the inbred Cleopatra III, and the third term is for paths through the non-inbred founders. Therefore, the inbreeding coefficient for Bernice III is 0.40625.

4. Compute the inbreeding coefficient of Cleopatra V.

   (a) The kinship between Bernice III and Ptolemy X Alexander I is

   $$1(2)(1+0)(1/2)^{3+1} + 1(2)(1+0.25)(1/2)^{3+1} + 2(8)(1+0)(1/2)^{6+1} = 0.40625.$$

   The first term is for the paths through the non-inbred Ptolemy VIII Physcon, the second term is for the paths through the inbred Cleopatra III, and the third term is for paths through the non-inbred founders. Therefore, the inbreeding coefficient for Cleopatra V is 0.40625.

5. Compute the inbreeding coefficient of Cleopatra VII.

   (a) This one is hard. We compute the kinship between Cleopatra V and Ptolemy XII. They have 5 common ancestors: Ptolemy IX, Cleopatra III, Ptolemy VIII, Ptolemy V, Cleopatra I. We consider the paths through each common ancestor.

   - Ptolemy IX: $(1)(1+0.25)(1/2)^{3+1}$
   - Cleopatra III: $(2)(1+0.25)(1/2)^{4+1} + (3)(1+0.25)(1/2)^{5+1}$
   - Ptolemy VIII: $(2)(1+0)(1/2)^{4+1} + (3)(1+0)(1/2)^{5+1}$
   - Cleopatra I: $(4 \cdot 2)(1+0)(1/2)^{4+3+1} + (4 \cdot 3)(1+0)(1/2)^{5+3+1}$

   The answer is $P_9 + C_3 + P_8 + 2 \times C_1 = 0.43359375$.

Jacquard's Δ values

You can verify the values for $\Delta_1, \Delta_2, \Delta_3, \Delta_9$ in the final row of the table below. Arguments for Δ_1 and Δ_3 are in the lecture slides. The argument for Δ_3 is the same as the argument for Δ_5. The arguments for Δ_4 and Δ_6 are the same. You can check the remaining calculations. Δ_7 and Δ_9 are the hardest.

RELATIONSHIP	Δ_1	Δ_2	Δ_3	Δ_4	Δ_5	Δ_6	Δ_7	Δ_8	Δ_9	ϕ_{ij}
Self	0	0	0	0	0	0	1	0	0	$\frac{1}{2}$
Parent-offspring	0	0	0	0	0	0	0	1	0	$\frac{1}{4}$
Half sibs	0	0	0	0	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{8}$
Full sibs/dizygotic twins	0	0	0	0	0	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$
Monozygotic twins	0	0	0	0	0	0	1	0	0	$\frac{1}{2}$
First cousins	0	0	0	0	0	0	0	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{16}$
Double first cousins	0	0	0	0	0	0	$\frac{1}{16}$	$\frac{6}{16}$	$\frac{9}{16}$	$\frac{1}{8}$
Second cousins	0	0	0	0	0	0	0	$\frac{1}{16}$	$\frac{15}{16}$	$\frac{1}{64}$
Uncle-nephew	0	0	0	0	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{8}$
Offspring of sib-matings	$\frac{1}{16}$	$\frac{1}{32}$	$\frac{1}{8}$	$\frac{1}{32}$	$\frac{1}{8}$	$\frac{1}{32}$	$\frac{7}{32}$	$\frac{5}{16}$	$\frac{1}{16}$	$\frac{3}{8}$